Searching for more simple lessons from complexity

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In memory of Leo P. Kadanoff
1937-2015
Leo was always searching for simple ways to understand complex systems

http://inspirehep.net/record/1283384/plots

Simple Lessons from Complexity
Nigel Goldenfeld and Leo P. Kadanoff

The complexity of the world is contrasted with the simplicity of the basic laws of physics. In recent years, considerable study has been devoted to systems that exhibit complex outcomes. This experience has not given us any new laws of physics, but has instead given us a set of lessons about appropriate ways of approaching complex systems.

Science, 284, 87-89 (1999)
Outline

• Some joint work with Leo on reversible Boolean networks
  – We looked for some simplicity in the dynamics of a specific class of dynamical systems (but found that properties were unexpectedly sensitive to certain details)

• Recent work done in the context of quantum computing that I think Leo would have liked.
Boolean networks (N-K models)


Each of N Ising elements $\sigma_i$ has K inputs

$$\sigma_i(t + 1) = f(\sigma_{j_1(i)}(t), \sigma_{j_2(i)}(t), \sigma_{j_3(i)}(t))$$

Time evolution determined by randomly chosen functions of randomly chosen inputs
Why study Boolean networks?

Can enumerate all possible networks and functions, so one could hope to do a quantitative statistical analysis and understand generic behavior quantitatively.

We investigated Boolean networks with reversible dynamics.

Usual Boolean network:
\[ \sigma_i(t + 1) = f(\sigma_{j_1(i)}(t), \sigma_{j_2(i)}(t), \sigma_{j_3(i)}(t)) \]

Reversible Boolean network:
\[ \sigma_i(t + 1) = f(\sigma_{j_1(i)}(t), \sigma_{j_2(i)}(t), \sigma_{j_3(i)}(t))\sigma_i(t - 1) \]

One question we asked: what is the distribution of orbit lengths (how many time steps do the orbits take before they repeat).

The distribution of orbit lengths of reversible Boolean networks is nontrivial!

Orbit lengths for intermediate $K$ are longer than for either small $K$ or large $K$. 

(”Normal” Boolean nets do not do this.)

Behavior arises from combination of symmetry and randomness

• A typical orbit closes because it hits a “symmetry point” where the time evolution reverses.

• But a small fraction of the random functions do not have symmetry points, and have very long orbits.
  (Fraction of functions with no symmetry points is $1/2^2^K$, which increases very quickly with K.)
Upshot

The behavior of the reversible Boolean networks is subtle, and small changes to the choice of functions can cause huge changes in the distribution of cycle lengths. (So behavior is not cleanly universal.)

It was a lot of fun to work with Leo to figure out the reasons for the unusual behavior.
Nonlinear dynamics of a strongly driven single spin solid state qubit


The work uses material from Leo’s course on nonlinear dynamics that he taught in the 1990’s.
The goal: single spin qubits in silicon/silicon-germanium heterostructures

Drive resonant electron spin transitions by applying strong electric field to an electron spin in a strong magnetic field gradient.

Obtain resonance at the Larmor frequency, $f = \hbar g \mu_B$.
Experimental setup

The goal: coherent manipulation of well-understood spin qubit

Strong microwave driving yields Rabi oscillations: coherent oscillations of spin state.

E. Kawakami et al., PNAS 113 11738-11743 (2016)
This talk is about an experiment where strong driving yields unexpected dependence of resonant frequency on power.
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At this power, there are 3 resonant frequencies.

P. Scarlino, E. Kawakami, et al.
Thibaut Jullien: Complex behavior could be due to frequency-dependent attenuation of RF lines

This work:

• Frequency-dependent attenuation is consistent with observed behavior

• If (1) resonant frequency is a known function of power, and (2) attenuation is a function of frequency, then frequency-dependent attenuation of the RF lines can be determined. (Experimentally very very useful!)
Set up situation as a problem of finding a fixed point of a dynamical system.

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Assume resonant frequency $f^*(I)$ is known function of $I$, $\mathcal{F}(I)$, where $I$ is the intensity of the drive at the sample.

Intensity at sample = $I\gamma(f(I))$; $I$ is the intensity at the source, and $\gamma(f)$ characterizes the attenuation.

$f^*$ satisfies:

$$f^*(I) = \mathcal{F}(I\gamma(f^*(I))) .$$

So we are are looking for a solution of a fixed point equation.
Mechanism leads naturally to three resonant frequencies at one value of the power.

\[ f^*(I) = \mathcal{F}(I\gamma(f^*(I))) \]

Plots for the choice \( \mathcal{F}(I\gamma(f)) = f_0 + I\gamma(f) \) and two different \( \gamma(f) \)'s.
If characteristics of nonlinear oscillator are known, can use this effect to measure frequency-dependent attenuation of microwave lines.

Accurate determination of the resonant frequency at low powers is important.
Summary

• If the resonant frequency of an oscillator is intensity-dependent, then frequency-dependent attenuation can yield multiple resonant frequencies at some powers.

• If the dependence of the resonant frequency on intensity is known, this provides a way to measure frequency-dependent attenuation in situ. Accurate measurements of resonant drive at low driving are critical for doing this.
Overall summary

I am greatly privileged to have had the opportunity to work with Leo and look for simple lessons in a variety of complex systems (even though not all the lessons turned out to be simple).