Turbulent Rayleigh-Bénard convection with polymers: Understanding how heat flux is modified

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Order and Chaos in Little Worlds:
Investigations of Reality in Computer Models
Q: What is an interesting problem?
A: One that helps us form powerful ideas

Q: How can one find a problem like this?
A: Find something beautiful and general that nobody understands

Q: How do you do that?
A: That is an interesting problem
It's an experience like no other experience I can describe, the best thing that can happen to a scientist, realizing that something that has happened in his or her mind exactly corresponds to something that happens in nature. One is surprised that a construct of one's own mind can actually be realized in the honest-to-goodness world out there. A great shock, and a great, great joy.

Chaos: Making a New Science by James Gleick
Turbulent Heat Flow: Structures and Scaling

For very many years, scientists have studied the motion of enclosed fluids heated from below and cooled from above. The containers for these Rayleigh–Bénard systems have ranged in size from soda cans to swimming pools. Usually, as a fluid is heated it will become less viscous and more buoyant, rising up to the surface and cooling down there. This causes the fluid to rotate to the left in the Northern Hemisphere and to the right in the Southern Hemisphere. Vertebrate blood circulation, on the other hand, is driven by differences in pressure rather than density differences.

Geometrical structures and scaling behavior provide insights into the nature of convective turbulence and some risky generalizations about “complex systems.”

Leo P. Kadanoff

Which was taken from a computer simulation of the surface of the Sun. That picture shows many cold plumes falling downward into the Sun. More mundane plumes can be seen in a wide variety of laboratory pictures and simulations. Now let us turn back to figure 1, which shows a...
The governing equations for Rayleigh-Bénard convection (without polymers):

\[
\frac{\partial u}{\partial t} + u \cdot \nabla u = -\frac{1}{\rho} \nabla p + \nu \nabla^2 u + \alpha g T \hat{z}
\]

\[
\frac{\partial T}{\partial t} + u \cdot \nabla T = \kappa \nabla^2 T
\]

\[
\nabla \cdot u = 0
\]

where \( u \) is the velocity field, \( T \) the temperature field, \( p \) the pressure, \( \rho \) the density, \( \hat{z} \) the unit vector in the vertical direction, and \( \alpha, \nu, \kappa \) are respectively the thermal expansion coefficient, kinematic viscosity, and thermal diffusivity of the fluid.
Exact balance:

\[ \varepsilon_u = \frac{K^3}{H^4}(\text{Nu} - 1)\text{Ra Pr} \]

\[ \varepsilon_T = K \frac{(\Delta T)^2}{H^2} \text{Nu} \]

\[ \text{Ra} = \frac{\alpha g \Delta TH^3}{\nu \kappa} \]  
\[ \text{Pr} = \frac{\nu}{\kappa} \]

\[ \text{Nu} = \frac{Q}{(k \Delta T / H)} \]
Earlier results

1. Polymer effects in the bulk without boundaries

Direct numerical simulation (DNS) of **homogeneous** turbulent thermal convection with polymers inside a box of height $H$ shows that heat flux is **enhanced** by polymers.


\[
\frac{D\vec{u}}{Dt} = -\nabla p + \nu_s \nabla^2 \vec{u} + \nabla \cdot \mathbf{T} + \alpha g \theta \hat{z}
\]

\[
\frac{D\theta}{Dt} = \kappa \nabla^2 \theta + \beta u_z
\]

where $\vec{u}$ and $\theta$ and $\theta = T - (T_0 - \beta z)$ satisfy periodic boundary conditions.
A common model of polymers (FENE-P):
A polymer chain is modeled by two massless beads connected by a spring.

\[ \bar{l} = \sqrt{3/\langle r^2 \rangle_{eq}} \bar{r} \]

\[ \bar{F}^{(i)}_{\text{drag}} = \zeta (\bar{u}^{(i)} - \bar{v}_{\text{bead}}^{(i)}) \ ; \ i = 1, 2 \]

\[ \bar{F}^{(1)}_{\text{spring}} = -\bar{F}^{(2)}_{\text{spring}} = C \left( \frac{b^2 - 3}{b^2 - \langle l^2 \rangle} \right) \bar{l} \ ; \ l = |\bar{l}| \]

\[ \bar{F}^{(2)}_{\text{drag}} + \bar{F}^{(2)}_{\text{spring}} - \bar{F}^{(1)}_{\text{drag}} - \bar{F}^{(1)}_{\text{spring}} + \text{force due to Brownian motion} = 0 \]

\[ \bar{u}^{(2)} - \bar{u}^{(1)} \propto \bar{l} \cdot \nabla \bar{u} \ ; \ \bar{v}_{\text{bead}}^{(2)} - \bar{v}_{\text{bead}}^{(1)} \propto \frac{d\bar{l}}{dt} \]

\[ \frac{d\bar{l}}{dt} \approx \bar{l} \cdot \nabla \bar{u} - \frac{1}{2\tau} \left( \frac{b^2 - 3}{b^2 - \langle l^2 \rangle} \right) \bar{l} + \text{term due to Brownian motion} \]

\( \tau = \text{relaxation time} \)

Peterlin approximation
The stress due to polymers is

\[ \mathbf{T}_{ij} = \frac{\nu_p}{\tau} \left( PR_{ij} - \delta_{ij} \right) \]

conformation tensor \( R_{ij} = \langle l_i l_j \rangle \)

\[ \nu_s + \nu_p = \text{zero-shear viscosity of the polymer solution} \]

\[ \nu_s = \text{solvent viscosity} \]

\[ \gamma = \nu_p/\nu_0 \text{ is a measure of polymer concentration} \]

\[
\frac{\partial R_{ij}}{\partial t} + \bar{u} \cdot \nabla R_{ij} = \frac{\partial u_i}{\partial r_k} R_{kj} + R_{ik} \frac{\partial u_j}{\partial r_k} - \frac{1}{\tau} \left( PR_{ij} - \delta_{ij} \right)
\]

In the limit of \( b \to \infty, P \to 1 \), the FENE-P model reduces to the Oldroyd-B model.
We have found an enhancement of heat transport as measured by the Nusselt number (Nu) for small $Wi = \tau \frac{U_{\text{rms}0}}{H}$.

$U_{\text{rms}0}$ is the rms velocity fluctuations in the absence of polymers.
A simple phenomenological theory:

Let \( \varepsilon_{tot} = \varepsilon_u + \varepsilon_p(Wi) = \frac{U_{rms}^3}{l_0} \); \( \varepsilon_T = \frac{U_{rms}T_{rms}^2}{l_T} \)

\[ \varepsilon_{tot} \sim \text{NuRa} \]

\[ \varepsilon_T \sim \text{Nu} \quad \text{and} \quad \text{Nu} \sim U_{rms}T_{rms} \quad \Rightarrow \quad T_{rms} \sim l_T \]

\( l_T \) increases with \( Wi \) but \( l_0 \) \sim unchanged

\[ l_0 \sim \text{constant} \quad \Rightarrow \quad U_{rms} \sim T_{rms}^{1/2} \]

\( \therefore \) \( \text{Nu} \sim T_{rms}^{3/2} \sim l_T^{3/2} \)
2. Experimental results at moderate Ra

Experimental studies found that polymers reduce heat transport for moderate Ra ~ $10^9$-$10^{10}$.


Wei, Nui and Xia, PRE 86, 016325 (2012)
Exact balance:

$$\mathcal{E}_u = \frac{\kappa^3}{H^4} (\text{Nu} - 1) \text{Ra Pr}$$

$$\mathcal{E}_T = \kappa \frac{(\Delta T)^2}{H^2} \text{Nu}$$

Both the bulk region and the boundary layers contribute to $\mathcal{E}_u$ and $\mathcal{E}_T$ with the contribution from the boundary layers dominating at moderate Ra.

S. Grossmann and D. Lohse, JFM 407,27 (2000)
3. Polymers in steady-state boundary layer flow

We have generalized the classical Prandtl-Blasius-Pohlhausen (PBP) boundary layer analysis to boundary layer flow with polymers.

\[ u_x \partial_x u_x + u_y \partial_y v_x = \nu_s \partial_{yy} u_x + \partial_y T \]

\[ u_x \partial_x T + u_y \partial_y T = \kappa \partial_{yy} T \]

Our generalized PBP boundary layer analysis shows that polymer stretching gives rise to a space-dependent effective viscosity

$$\nu_{\text{eff}}(\xi)$$

with

$$\xi \equiv \sqrt{\frac{U}{v_0 x y}}$$

and it can be found self-consistently by solving the equations of motion.

The change in Nu can thus be obtained as a function of

$$Wi = \frac{U \tau}{H}$$

and $$\gamma$$.
\[ g(\xi) = \frac{v_s + \nu_{\text{eff}}(\xi)}{\nu_0} \]

\[ \text{Wi} = 5 \quad \gamma = 0.2 \]

\[ \text{Nu}_0 \text{ is the value for a Newtonian fluid of kinematic viscosity } \nu_0 \]

Qualitative agreement with experimental results!

Wei, Nui and Xia, PRE 86, 016325 (2012)
Most Recent Study

4. Direct numerical simulations at moderate Ra

R. Benzi, ESCC, E. DeAngelis

\[ \frac{D\vec{u}}{Dt} = -\nabla p + \nu_s \nabla^2 \vec{u} + \nabla \cdot \mathbf{T} + \alpha g (T - T_0) \hat{z} \]

\[ \frac{DT}{Dt} = \kappa \nabla^2 T \]

- No-slip boundary condition for velocity and fixed temperature difference of \( \Delta T \) at top and bottom plates
- Periodic boundary conditions in the lateral \( x \) and \( y \) directions
- Oldroyd-B polymers of different relaxation times \( \tau \) are studied

Ra = 2.1 x 10^5 \quad Pr = 7
Results:

Heat flux is enhanced and the amount of enhancement first increases then decreases with Wi.

\[ Wi = \tau \frac{U_c}{H} ; \quad U_c = \sqrt{\alpha g \Delta TH} \]

Wi=0 corresponds to Newtonian flow without polymers

\text{Nu}_s \text{ is the value for Newtonian flow without polymers}

Heat flux is \textit{enhanced} and the amount of enhancement first increases then decreases with Wi.
In the presence of polymers, exact balance becomes:

\[ \varepsilon_u + \varepsilon_p = \frac{K^3}{H^4} (\text{Nu} - 1) \text{Ra Pr} \]

\[ \varepsilon_p = \left\langle -u_a \partial_b T_{ab} \right\rangle = \left\langle E_p \right\rangle \]

Polymers affect scales of flow smaller than or equal to the Lumley scale. The Lumley scale increases with Wi and when it approaches the system size, polymers interfere with the convective instability causing \( \varepsilon_u \) to decrease.
Negligible amount of polymer stretching for Wi=2.

The amount of polymer stretching increases significantly as Wi is increased.

Polymer stretching extends into the bulk of the flow, and reaches maximum at the center of the cell.
Our aim is to provide a theoretical explanation of the change in $\text{Nu}$ due to polymers when $\varepsilon_u$ is held fixed.

$$\text{Nu} \gg 1 \text{ and } \varepsilon_p \approx 0 \text{ at } Wi=2 :$$

$$\frac{\varepsilon_p}{\varepsilon_{u0}} = \frac{\text{Nu}}{\text{Nu}_0} - \frac{\varepsilon_u}{\varepsilon_{u0}} \equiv \left( \frac{\delta \text{Nu}}{\text{Nu}_0} \right)_{\varepsilon_u \text{ fixed}}$$

$\text{Nu}_0 = \text{Nu} \text{ at } Wi=2$

$\delta \text{Nu} = \text{Nu}-\text{Nu}_0$

Our basic idea: for fixed $\varepsilon_u$, the relative change in $\text{Nu}$ can be calculated using the generalized PBP boundary layer theory

$$\left( \frac{\delta \text{Nu}}{\text{Nu}_0} \right)_{\varepsilon_u \text{ fixed}} \sim \left( \frac{\delta \text{Nu}}{\text{Nu}_0} \right)_{\text{PBP}}$$
We estimate the effective viscosity due to polymers from the DNS data using

\[ \frac{\nu_{\text{eff}}(z)}{\nu_s} = \frac{E_p(z)}{\varepsilon_u(z)} \]

\[ g(\xi) = \frac{\nu_s + \nu_{\text{eff}}(\xi)}{\nu_0} \]
Heat flux is enhanced when $\xi_m$ lies outside the thermal boundary layer.
\[
\left( \frac{\delta \text{Nu}}{\text{Nu}_0} \right)_{\varepsilon_u \text{ fixed}} = \frac{\text{Nu}}{\text{Nu}_0} - \frac{\varepsilon_u}{\varepsilon_{u0}}
\]

is calculated from generalized PBP boundary layer theory using the fitted form of
\[
g(\xi) = A\{ 1 - \exp[-(\xi / \xi_0)^{\alpha}] \} \]
from DNS data.
Conclusions

• Using direct numerical simulations, we find that polymer enhances heat flux for Wi>2, and the amount of enhancement first increases then decreases with Wi.

• The non-monotonic behavior of heat flux enhancement is the combined effect of a decrease in $\varepsilon_u$ and an increase in $\varepsilon_p$ with Wi.

• We have shown that for fixed $\varepsilon_u$, the change in Nu due to polymers can be computed using the generalized boundary layer theory supplemented by the effective viscosity profile estimated from the DNS data.

• Our work thus provides a physical way to understand the modification of heat flux by polymers in turbulent Rayleigh-Bénard convection.
Thank you for your attention!

